Matrices: Theory & Applications
Errata & comments

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See also the additional list of exercises on
http://www.umpa.ens-lyon.fr/~serre/extra.pdf,
and the solutions to the exercises on

Errata for the first edition

1. Page 17. In the last two displayed identities, the signs in the right-hand side should not be there.

2. In Exercise 19, page 35, question c), read $X^{j-1}$ instead of $X^j$.

3. The discussion page 44, about the distance between spectra, could be put in a more rigorous style. If $n \geq 1$, let us consider an equivalence relation on $\mathbb{C}^n$, defined by $x \mathcal{R} y$ if and only if the coordinates of $y$ are obtained from those of $x$ by a permutation. Let denote by $X_n$ the quotient of $\mathbb{C}^n$ by this relation. The set $X_n$ differs from the set $\mathbb{C}^n$ of $n$-uplets, because its elements are non-ordered $n$-uplets. It also differs from the set of subsets of $\mathbb{C}$ of cardinality $n$, because elements such that $(x,x,y)$ and $(x,y,y)$ could be confused in the latter (and have only two elements), though they are distinct in $X_3$.

The spectrum of an $n \times n$ matrix is clearly an element of $X_n$, since we do not pay attention to the order in which we write the eigenvalues, but we do care about their multiplicities. More generally, the set of the zeroes of a polynomial of degree $n$ is an element of $X_n$.

The distance $d$ is defined on $X_n$. It is obtained by taking the infimum of the $\ell^\infty$-distance between all the realizations of $A, B \in X_n$ as $n$-uplets.

4. Corollary 3.1.1, page 45, is obviously false. For instance, $0_n$ is diagonal, but every neighbourhood contains non trivial strictly upper triangular matrices. Such ones cannot be diagonalizable since their spectrum reduces to \{0\}.
One could say that the set of matrices with simple eigenvalues (they are diagonalizable, at least when $k = \mathbb{C}$), is open. One could as well say that the set of diagonalizable matrices is dense in $M_n(\mathbb{C})$.

5. Page 45, Section 3.2, read $\|Mx\|_2 = \|M^*\|_2$, instead of $\|Ax\|_2 = \|A^*\|_2$, and $MM^* - M^*M$ instead of $AA^* - A^*A$.

6. In Theorem 3.2.1, page 46, we have actually an equivalence.

7. The proof of Theorem 4.5.1, page 72, though essentially correct, misses some topological issues. A connected component $G$ of $\mathcal{G}$, a compact set of the plane, might not be simply connected. In such a case, one of its holes could contain an other connected component $G'$ (or several ones). Then every Jordan curve encircling $G$ will encercle such $G'$. The proof given in the book computes the number $N(G)$ of eigenvalues belonging to $G$ and to every such $G'$. However, this is sufficient to conclude, using the following argument. Say that a connected component $G$ is of level 0 if its holes do not contain any other component, and that it is of level $\ell$ if its holes contain a component of level $\ell - 1$, and every component contained in the holes of $G$ is of level at most $\ell - 1$. When $G$ is of level 0, the searched number is just $N(G)$. Then we compute inductively: The number of eigenvalues in a component $G$ of level $\ell$ equals $N(G)$, minus the sum of the $N(G')$'s, over all the components $G'$ contained in the holes of $G$, these being of level at most $\ell - 1$.

8. Page 83, in the proof of Theorem 5.3.1, the case $x > 0$ and $\|x\|_1 = 1$ actually yields $Ax > 0$, which is better than stated.

9. Page 100, right next to the proof of Proposition 6.1.3, read “the quadratic ring $\mathbb{Z}[\sqrt{14}]$ is a principal ideal domain, though not Euclidean.”


12. In Theorem 6.3.2, page 104, the first sentence is incomplete. Read “with $A_0$, $A_1 \in \text{GL}_n(k)$”.

13. Exercise 5 of Chapter 6, “Lagrange’s Theorem” is due to Wedderburn when $m = 1$ and to Guttman in the general case.


15. Page 119, between (7.5) and (7.6), read $\text{Sp}(M^i) = \log \text{Sp}(H^i)$, instead of $\text{Sp}(M^i) = \log \text{Sp}(M^i)$.

16. In the proof of Lemma 7.4.2, read “The arcwise-connected set $A$ contains the origin and the point $z = 1$, since...”
17. Pages 127–128, the computation of $\mathbf{SP}_n \cap \mathbf{O}_{2n}$ is much simpler if we write that $M^T = M^{-1}$ for $M \in \mathbf{O}_{2n}$. Then $M \in \mathbf{Sp}_n$ means $M^T J M = J$, that is $JM = MJ$, which gives immediately $D = A$ and $C = -B$. The same idea may be used in Sections 7.4 and 7.5.

18. Page 128, read “the map $M \mapsto A + iB$ is an isomorphism from $\mathbf{SP}_n \cap \mathbf{O}_{2n}$ onto $\mathbf{U}_n$.”

19. For a general field of scalars, Corollary 7.6.1 is true and follows from Theorem 2.3.1.

20. Page 130, read “$(\ker M)^+$, a subspace of dimension $r_i$.”

21. Page 141, read “after summation from $k = 0$ to $l$.”

22. In Proposition 8.1.3, one can take the slightly better

\[ d_0 = 4 + \frac{3c_0 2^\alpha}{2^{\alpha-1} - 1}. \]


24. An easier proof of uniqueness in Proposition 8.3.1: once we have $Q = R$, with $Q \in \mathbf{U}_n$ and $R$ upper triangular with a positive real diagonal, just say that $R^{-1} = Q^{-1} = Q^T = R^T$, where $R^{-1}$ is upper and $R^T$ is lower triangular. Therefore $R^T$, or $R$, is diagonal. Then $r_{jj} \in (0, +\infty)$ and $|r_{jj}| = 1$ give $r_{jj} = 1$, hence $R = I_n$.

25. Page 158, line 14, read “the case $\Delta(\rho(J)) \geq 0$”.

26. Page 161, read “Hence $P(0) \neq 0'$ instead of “Hence $P(0) \neq 1$”.

27. Page 164, line 6. Read $p_k = r_k + \cdots$ instead of $p_{k+1} = r_k + \cdots$.

28. Page 164, two lines after (9.8). Read $n\tau_{CG} \gg 1$ instead of $n\tau_{CG} \ll 1$.

29. In Theorem 10.1.1, the complexity is not $3n^3 + O(n^2)$, but $\frac{16}{3}n^3 + O(n^3)$.

30. Page 171, In the displayed formula for $X_\pm$, $\bar{q}$ should be $q$.

31. Page 172. Formula (10.1) should be

\[ P_{j+1}(X) = (X - m)P_j(X) - |a|^2P_{j-1}(X). \]

32. Page 173, in the description of Sturm sequences, one should add the requirement that if $Q_0(c) = 0$, then $(x - c)Q_0(x)Q_1(x) > 0$ for $x$ close to $c$.

33. Page 173, penultimate line. Read $A_0$ instead of $A_1$.

34. Page 176, read “Finally, the complexity of the QR iteration on a Hessenberg matrix”, instead of factorization of.

36. Page 179, read “subspace \([e^1, \ldots, e^{r-1}]\) spanned by \(e^1, \ldots, e^{r-1}\)” (replace \(e^{n-p}\) by \(e^{r-1}\)).

37. Page 181, line 13. The angles \(\theta\) and \(\theta + \pi\) give matrices \(K\) and \(K_+\) differing from each other by signs. Actually, \(K_+\) and \(K\) are conjugated through the symmetry

\[ x \mapsto (x_1, \ldots, x_{p-1}, -x_p, x_{p+1}, \ldots, x_{q-1}, -x_q, x_{q+1}, \ldots, x_n). \]

Such sign changes just propagate, but do not alter the method in a significant way. One can prove that adding \(\pi\) to some of the angles has the effect to conjugate the iterates by a matrix \(P\Sigma\), where \(P\) is a permutation matrix and \(\Sigma\) is a diagonal matrix of signs. Due to the conjugation by \(P\), the pairs \((p, q)\) are modified. However these modifications are harmless; for instance, the range of \((i, j) \mapsto |h_{ij}|\) remains the same. This is why we may restrict to angles \(\theta \in (-\pi/2, \pi/2]\).

38. Page 183, read \(k_{pp} - h_{pp} = -th_{pq}\) instead of \(k_{pp} - h_{pp} = th_{pq}\).

39. In Theorem 10.3.2, page 183, we interpret (as other authors do) the estimate \(\|E_{k+N}\| \leq c\|E_k\|^2\) for \(N : = n(n-1)/2\) as indicating that the Jacobi method is of order two, at least when the matrix has simple eigenvalues. If we say that a method is of order \(\alpha > 1\) when the error \(\epsilon_{k+1}\) at step \(k + 1\) is roughly \(\epsilon_k^\alpha\), then the estimate rather tells that Jacobi’s method is of order \(2^{1/N}\). It is however valuable that this order is strictly larger than one. See the Additional Exercise 197 for an analysis of the exact order when \(n = 3\) (slightly better than \(2^{1/3}\)).

When performing the Jacobi method with a cyclic choice of the pair \((p, q)\) (instead of the optimal choice), it is reasonable to speak of the order per sweep. A sweep is a set of \(N\) consecutive iterations. Along a sweep, every off-diagonal entry has been settled to zero exactly once. The analysis of the order per sweep is still a research topic. For matrices with simple eigenvalues, this order is at least two and might be three. Thus the Jacobi method could be comparable with the QR algorithm.

40. Page 186, read “is stable under \(M\)” instead of “is stable under \(A\)”.

**Errata for the second edition**

1. Page 109, it is claimed that \(H_n \cdot H_n = \text{M}_n(C)\). This is false. Obviously if \(n = 1\), because then \(H_n = \mathbb{R}\). More generally, one always have \(\text{Tr}(HK) \in \mathbb{R}\). It turns out (Horn & Johnson, vol. I, Thm 4.1.7) that \(H_n \cdot H_n\) is the set of complex matrices that are conjugated to a real matrix.

The correct statement is \(\text{Sym}_n(\mathbb{R}) \cdot \text{Sym}_n(\mathbb{R}) = \text{M}_n(\mathbb{R})\).